COMPARING QUANTITIES (ON AN EXPRESSION MAT) 6.1.1 – 6.1.2

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for simplifying (and later solving) inequalities and equations.

Tiles may be removed or moved on the mat in the following ways:

(1) Removing the same number of opposite tiles (zeros) on the same side;

(2) Removing an equal number of identical tiles (balanced set) from both the left and right sides;

(3) Adding the same number of opposite tiles (zeros) on the same side; and

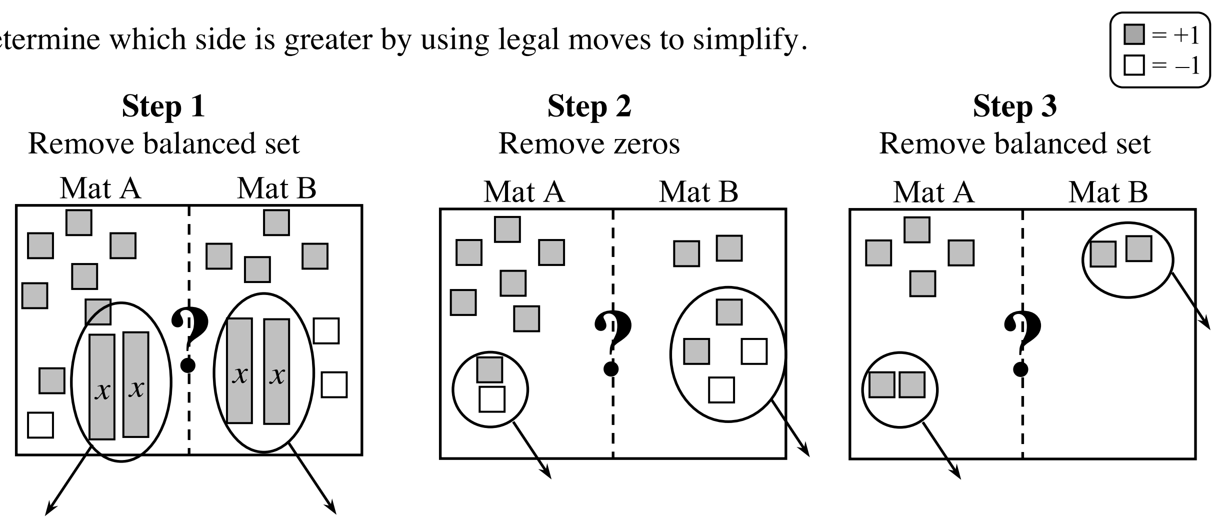
(4) Adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called "legal moves."

After moving and simplifying the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

Example 1

Determine which side is greater by using legal moves to simplify.



The left side is greater because after Step 3: $4 > 0$. Also, after Step 2: $6 > 2$. Note that this example shows only one of several possible strategies.

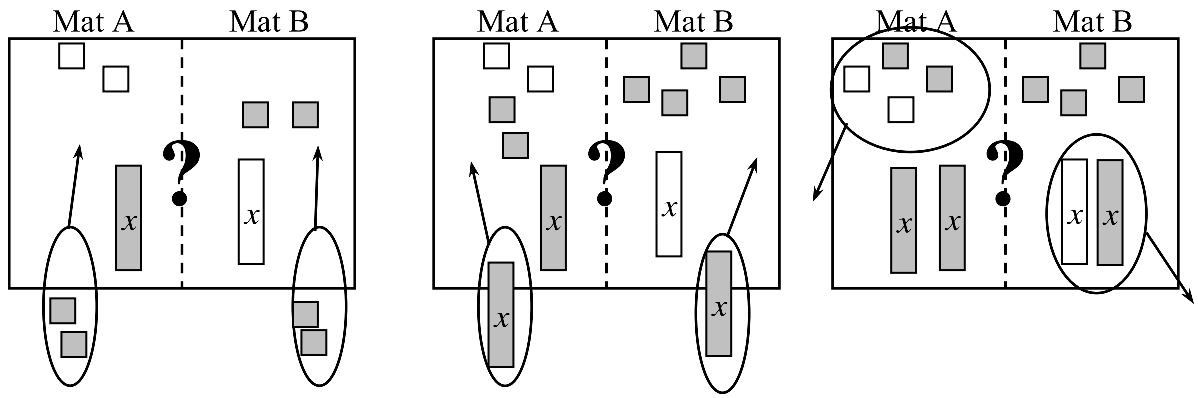
Example 2

Use legal moves so that all the x-variables are on one side and all the unit tiles are on the other.

Step 1 Add balanced set

Step 2 Add balanced set

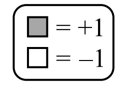
Step 3 Remove zeros



What remains is $2x$ on Mat A and 4 on Mat B. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of "x." Students are expected to record the results algebraically as directed by the teacher. One possible recording is shown at right.

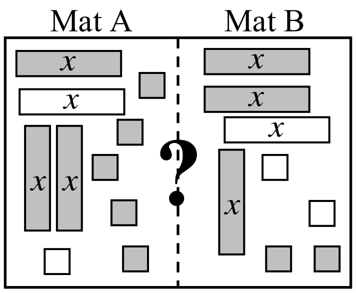
Mat A	Mat B
$x-2$	$-x+2$
$x-2+2$	$-x+2+2$
$x+x-2+2$	$-x+x+2+2$
$2x$	4

Problems



For each of the problems below, use the strategies of removing zeros or simplifying by removing balanced sets to determine which side is greater, if possible. Record your steps.

1.



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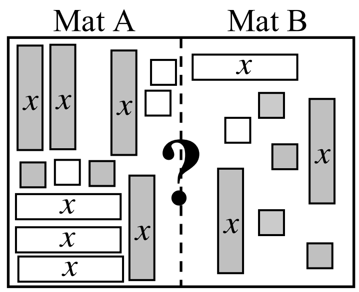

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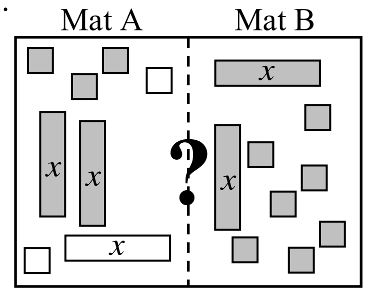


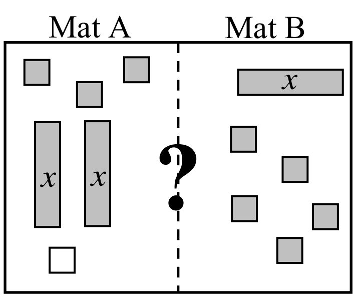
4. Mat A: $5+(-8)$
   Mat B: $-7+6$

5. Mat A: $2(x+3)-2$
   Mat B: $4x-2-x+4$

6. Mat A: $4+(-2x)+4x$
   Mat B: $x^2+2x+3-x^2$

For each of the problems below, use the strategies of removing zeros or adding/removing balanced sets so that all the x-variables are on one side and the unit tiles are on the other. Record your steps.

7.



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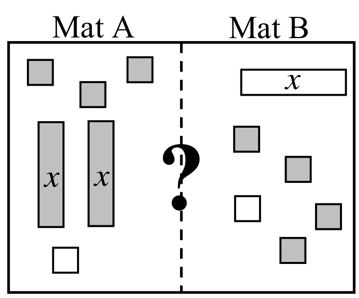

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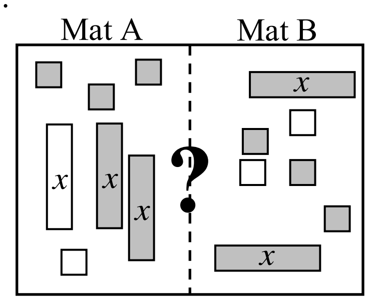


10. Mat A: $3x-2$
    Mat B: $2x+1$

11. Mat A: $4x+2+(-5)$
    Mat B: $2x+3+(-8)$

12. Mat A: $2x+3$
    Mat B: $-x-3$

Answers (Answers to problems 7 through 12 may vary.)

1.	A is greater	2.	B is greater	3.	not possible to tell
4.	B is greater	5.	not possible to tell	6.	A is greater
7.	A: x; B: 3	8.	A: $3x$; B: 1	9.	A: 1; B: x
10.	A: x; B: 3	11.	A: $2x$; B: –2	12.	A: $3x$; B: –6

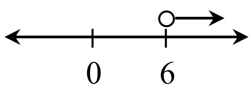
GRAPHING AND SOLVING INEQUALITIES 6.1.3 – 6.1.4

GRAPHING INEQUALITIES

The solutions to an equation can be represented as a point (or points) on the number line. If the expression comparison mat has a range of solutions, the solution is expressed as an inequality represented by a ray or segment with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution (≤ or ≥), while the open dot indicates that it is not part of the solution (< or >).

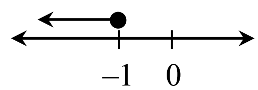
Example 1

$x > 6$



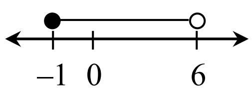
Example 2

$x \leq -1$



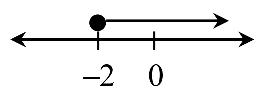
Example 3

$-1 \leq y < 6$



Example 4

$y \geq -2$

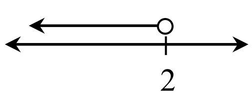


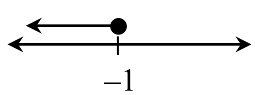
Problems

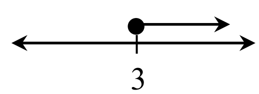
Graph each inequality on a number line.

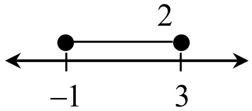
1. $m < 2$
2. $x \leq -1$
3. $y \geq 3$
4. $-1 \leq x \leq 3$
5. $-6 < x < -2$
6. $-1 < x \leq 2$
7. $m > -9$
8. $x \neq 1$
9. $x \leq 3$

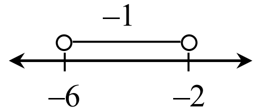
Answers

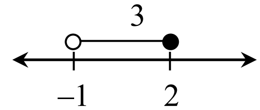
1. 

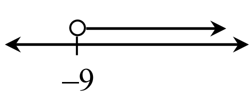
2. 

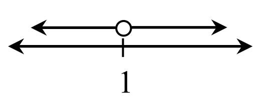
3. 

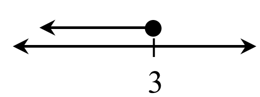
4. 

5. 

6. 

7. 

8. 

9. 

SOLVING INEQUALITIES

To solve an inequality, examine both of the expressions on an expression comparison mat. Use the result as a dividing point on the number line. Then test a value from each side of the dividing point on the number line in the inequality. If the test number is true, then that part of the number line is part of the solution. In addition, if the inequality is $\geq$ or $\leq$, then the dividing point is part of the solution and is indicated by a solid dot. If the inequality is $>$ or $<$, then the dividing point is not part of the solution, indicated by an open dot.

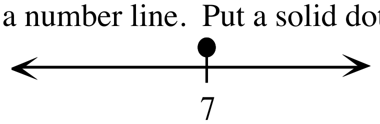
For additional information, see the Math Notes box in Lesson 6.1.4 of the *Core Connections, Course 2* text.

Example 1

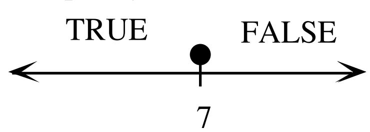
$9 \geq m + 2$

Solve the equation: $9 = m + 2$
$7 = m$

Draw a number line. Put a solid dot at 7.

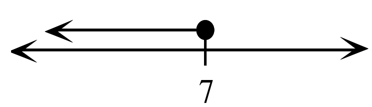


Test a number on each side of 7 in the original inequality. We use 10 and 0.



$m = 0$	$m = 10$
$9 > 0 + 2$	$9 > 10 + 2$
$9 > 2$	$9 > 12$
TRUE	FALSE

The solution is $m \leq 7$.

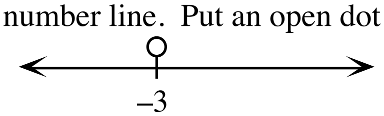


Example 2

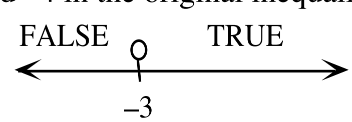
$-2x - 3 < x + 6$

Solve the equation:
$$\begin{aligned} -2x - 3 &= x + 6 \\ -2x &= x + 9 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

Draw a number line. Put an open dot at –3.

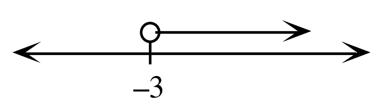


Test 0 and –4 in the original inequality.



$x = -4$	$x = 0$
$-2(-4) - 3 < -4 + 6$	$-2(0) - 3 < 0 + 6$
$8 - 3 < 2$	$-3 < 6$
$5 < 2$	TRUE
FALSE	

The solution is $x > -3$.



Problems

Solve each inequality.

1. $x + 3 > -1$	2. $y - 3 \le 5$	3. $-3x \le -6$
4. $2m + 1 \ge -7$	5. $-7 < -2y + 3$	6. $8 \ge -2m + 2$
7. $2x - 1 < -x + 8$	8. $2(m + 1) \ge m - 3$	9. $3m + 1 \le m + 7$

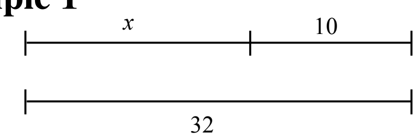
Answers

1. $x > -4$	2. $y \le 8$	3. $x \ge 2$	4. $m \ge -4$	5. $y < 5$
6. $m \ge -3$	7. $x < 3$	8. $m \ge -5$	9. $m \le 3$	

Initially, equations are solved either by applying math facts (for example, $4x = 12$, since $4 \cdot 3 = 12$, $x = 3$) or by matching equal quantities, simplifying the equation, and using math facts as shown in the examples below. Equations are often written in the context of a geometric situation.

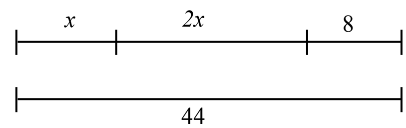
Write an equation that represents each situation and find the value of the variable.

Example 1



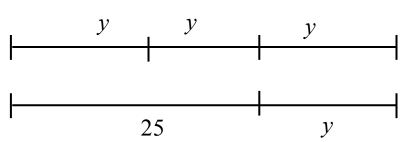
$$x + 10 = 32$$
$$x = 22$$

Example 2



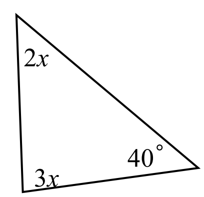
$$x + 2x + 8 = 44$$
$$x + 2x = 36$$
$$3x = 36$$
$$x = 12$$

Example 3



$$3y = 25 + y$$
$$2y = 25$$
$$y = 12.5$$

Example 4

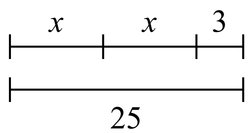


$$2x + 3x + 40 = 180$$
$$2x + 3x = 140$$
$$5x = 140$$
$$x = 18$$

Problems

Write an equation that represents each situation and then find the value of the variable.

1.



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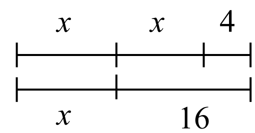

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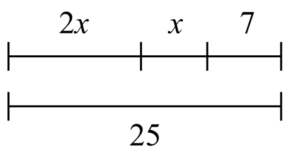

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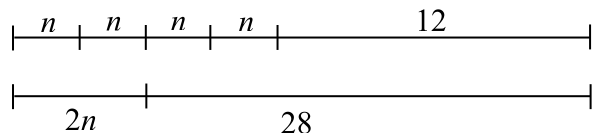

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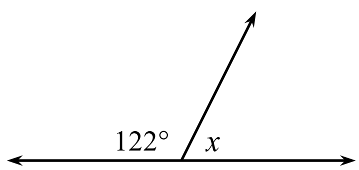

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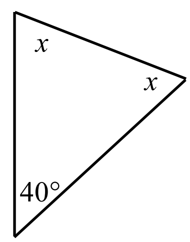

Solve each equation.

7. $x+7=-9$

8. $y-2=-3$

9. $-3y=24$

10. $\frac{m}{2}=-6$

11. $3x+2=11$

12. $4x+x+5=25$

13. $m+2m+7=m+11$

14. $x+9+x+x=30$

15. $3-y=9$

16. $4k+1=-7$

17. $x+3x+x+7=52$

18. $3m+7=m+11$

19. $2(y+3)=-12$

20. $3(c+2)+c+1=57$

Answers

1. $2x+3=25;\ x=11$
2. $2x+4=x+16;\ x=12$
3. $3x+7=25;\ x=6$
4. $4n+12=2n+28;\ n=8$
5. $122+x=180;\ x=58^\circ$
6. $2x+40=180;\ x=70^\circ$
7. $x=-16$
8. $y=-1$
9. $y=-8$
10. $m=-12$
11. $x=3$
12. $x=4$
13. $m=2$
14. $x=7$
15. $y=-6$
16. $k=-2$
17. $x=9$
18. $m=2$
19. $y=-9$
20. $c=12.5$